Capillary forces on millimeter-scale sediment particles: Experimental measurements and theoretical estimations

Nirmalya Chatterjee¹, Sergey Lapin² and Markus Flury¹

¹Dept. of Crop and Soil Sciences, Puyallup, Washington State University; ²Dept. of Mathematics, Pullman, Washington State University

Goal and Objective

The overall objective of this project is to understand the mechanism of transport of contaminants, both direct transport and facilitated transport by colloids, in the vadose zone. The specific objective is to measure the capillary forces on mm-scale particles due to moving air-water interfaces. The particles were obtained from the vadose zone under the waste tanks at the Hanford Site in Washington State. We then compare the experimentally measured forces with those obtained using theoretical calculations. We hypothesize that:

- Capillary forces are best approximated by assuming particles to be ellipsoidal in shape, rather than a spherical shape.
- Surface roughness and irregularity of natural sediment particle shapes enhance capillary forces by considerable amounts leading to increased mobilization of particles.
- Calculations on mm-scale particles can be extrapolated to colloidal scale particles which have been implicated in facilitated contaminant transport.

Background

- The Hanford Nuclear Site was established in 1943 as a Plutonium production facility for the US nuclear weapons production.
- Millions of gallons of low-level radioactive and hazardous waste (caustic solutions from dissolved fuel rods, sludge and solid wastes) have been dumped into the ground.
- Mobility of radioactive wastes in the semiarid soils of the Hanford geologic formation is expected to be limited because of slow vertical water movement in the soils.
- Laboratory studies using Hanford sediments indicated colloidal facilitated radioactive contaminant transport in the soils as a major cause of contaminant mobilization – in both saturated and unsaturated soil conditions. Such potential transport events may occur during strong infiltration conditions like localized thunderstorms and Chinook wind (warm and humid coastal winds from the Pacific Ocean) induced rapid snow-melts.
- Capillary forces and DLVO forces (electrostatic and van der Waals) are the most important forces acting on colloidal size particles during mobilization.

Hanford Site

- Hanford tanks under construction, 1940s
- Aerial view of the Hanford Site with the Columbia River in the background

Theory

For axisymmetric particles, the capillary rise, \( h \), the horizontal distance of the three-phase contact line from the \( z \)-axis, \( x \), and the angle of inclination of the curved interface from the undisturbed interface, \( \theta \), is:

\[
\frac{R}{\sin \theta} = \frac{k(h) - k(x)}{1 - k(x)} + \frac{k'(x)}{1 - k(x)}
\]

where the functions \( k(x) \) and \( k'(x) \) are the modified Bessel functions of the second kind of orders 0 and 1 and respectively:

- The force balance (net force \( f \)) on an axisymmetric particle is:
  \[
f = f_p + f_s + f_e + f_h
\]
  where \( f_p \) is the surface tension force, \( f_s \) is the buoyancy force, and the \( f_h \) is the hydrostatic pressure force.
- The pinning can be described by the Gibbs extension of the Young-Laplace equation:

\[
f_p = \gamma \cos \theta
\]

where \( \theta \) is the equilibrium contact angle, \( \theta \) is the actual contact angle, and \( \alpha \) is the wedge angle at the pinned contact line.

For non-axisymmetric particles, the capillary rise, \( h \), the horizontal distance of the three-phase contact line from the \( z \)-axis, \( x \), and the angle of inclination of the curved interface from the undisturbed interface, \( \theta \), is:

\[
\frac{R}{\tan \theta} = \frac{k(h) - k(x)}{1 - k(x)} + \frac{k'(x)}{1 - k(x)}
\]

where the functions \( k(x) \) and \( k'(x) \) are the modified Bessel functions of the second kind of orders 0 and 1 and respectively:

- The force balance (net force \( f \)) on a non-axisymmetric particle is:
  \[
f = f_p + f_s + f_e + f_h
\]
  where \( f_p \) is the surface tension force, \( f_s \) is the buoyancy force, and the \( f_h \) is the hydrostatic pressure force.
- The pinning can be described by the Gibbs extension of the Young-Laplace equation:

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where \( \theta \) is the equilibrium contact angle, \( \theta \) is the actual contact angle, and \( \alpha \) is the wedge angle at the pinned contact line.

Experimental Methods

Ten sediment particles with different shapes were used to study capillary forces.

- Schematic of a force-position curve obtained during immersion of a sediment particle. \( \alpha \) is the unconstrained position of the particle \( x \)-axis is the air-water interface, \( \gamma \) is the capillary forces on the particle due to the air-water interface as measured on the microbalance.
- Frames from 1000fps movies of 3D printed poly-acrylate particles showing different phenomena: (a) advancing \((1-\text{immersion})\) and receding \((2\text{-emersion})\) contact angles with a sphere \( (\text{b}) \) bottom and \( (\text{c}) \) top \( \alpha \) edge pinning of air-water interface with a circular cylinder.

Conclusion

For axisymmetric particles, the capillary rise, \( h \), the horizontal distance of the three-phase contact line from the \( z \)-axis, \( x \), and the angle of inclination of the curved interface from the undisturbed interface, \( \theta \), is:

\[
\frac{R}{\sin \theta} = \frac{k(h) - k(x)}{1 - k(x)} + \frac{k'(x)}{1 - k(x)}
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where \( \theta \) is the equilibrium contact angle, \( \theta \) is the actual contact angle, and \( \alpha \) is the wedge angle at the pinned contact line.

Theoretical Results

Table 2: Comparison of maximum capillary forces (Experimental and Theoretical)

<table>
<thead>
<tr>
<th>Particle</th>
<th>Immersion force (mN)</th>
<th>Emersion force (mN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTFE sphere</td>
<td>3.72</td>
<td>1.01</td>
</tr>
<tr>
<td>PTFE disk</td>
<td>3.68</td>
<td>1.00</td>
</tr>
<tr>
<td>Basalt</td>
<td>1.15</td>
<td>0.47</td>
</tr>
<tr>
<td>Granite</td>
<td>1.01</td>
<td>0.37</td>
</tr>
<tr>
<td>Hematite</td>
<td>0.89</td>
<td>0.36</td>
</tr>
<tr>
<td>Quartz</td>
<td>0.71</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Comparisons of experimental and theoretically calculated immersion curves assuming a spherical, ellipsoidal and cylindrical shape. Normalized dimensions of a quartz particle.

Implications

- Surface roughness and irregular shapes considerably increase capillary forces experienced by sediment particles, and such effects are even more important for colloids.
- Colloidal mobilization by air-water interfaces caused by water movement in soils are thus affected by particle shape and surface properties.
- Both non-axisymmetric particles and colloids containing contaminants can be mobilized during infiltration and drainage.
- Enhanced mobilisation of contaminants will be important to consider for the clean-up efforts at the Hanford Site.

Definitions

- **Vadose Zone**: The zone above the water table that is partially saturated with water.
- **Colloidal Mobilization**: The movement of minute particles (colloids) in water due to physical and chemical forces.
- **Surface Roughness**: The irregularities on the surface of a solid object that can affect capillary forces.

Publications